THE SKIN EFFECT AND FLUX DISTRIBUTION OF CONDUCTORS IN PROXIMITY TO IRON MASSES.*

By A. Press.

SYNOPSIS.

Insufficiency of Arnoldian Theory of Slot Self-induction.—The hitherto accepted method for estimating self-induction of conductors in proximity to iron, which assumes that internal self-induction flux is short-circuited by the neighboring iron, so that in slot wound conductors all the internal self-induction flux passes across the slot, is disproved by test with a small exploring compass needle. It is more nearly true that the internal self-induction flux distribution of a conductor is much the same as in air and that practically it is only the external self-induction flux that takes advantage of the proximity of iron masses.

Comparison of A. B. Field's and the Author's Solution on the Basis of the Arnoldian Theory.—A correction needs to be introduced into A. B. Field's mathematical solution for the slot wound conductor problem. The two boundary conditions should be: first, that the value of H at the top of the mth conductor is due to the m conductors carrying the load current; and, secondly, that the value of H at the bottom of the mth conductor on the basis of the Arnoldian theory should be due to $(m - 1)$ current carrying conductors. When these boundary conditions are properly introduced, it is found that the skin-effect factors are very different to those derived according to the A. B. Field requirement in which one of the boundary conditions above referred to is replaced by the condition merely that the mth conductor shall carry the total line current.

Consequences of the Newer Theory.—For purposes of estimating the skin effect of conductors in iron slots, or near iron masses, results more consistent with facts will be obtained if the author's solution for the skin effect is massed rectangular conductors in air is assumed. However, in estimating the external self-induction the Arnoldian theory will still apply.

IN the number of the PHYSICAL REVIEW for October, 1916, the writer gave the solution of the "Skin Effect" problem of massed rectangular conductors in air. At that time it was not thought that the above solution really applied equally well to the problem of the skin effect of conductors occurring in the open type of slots usual in armature constructions. It will be shown however that the "in air" solution above given is equally applicable to the case of Ruhmkorff coil constructions as well as to the analogous cases of core and shell type transformers and generally speaking to all cases where electrical windings are brought into proximity to iron masses.

For low as well as high frequencies it has become classic to assume that the self-induction flux of conductors placed near an iron surface enters the iron and on emerging penetrates the conductors parallel to such iron surface. It has been for this reason that Sir J. J. Thomson's formula for copper strip (see Recent Researches in Electricity and Magnetism, page 281) has been invoked. In the case of slot wound conductors A. B. Field's formulas have been employed (see Proc. A. I. E. E., 24, 659, 1905). The method of assuming that at the bottom of the slot the magnetic intensity is zero occurs in all the literature (see for example E. Arnold, Die Gleichstrommaschine). As a matter of fact the flux within the conductor is never magnetically short circuited by the iron as the experiments referred to below will clearly indicate. It is for this reason that whenever current carrying conductors are placed near an iron surface of whatever type, it would be much more exact to say that the flux always circulates about the conductors as if they were free in air. Thus, *iron even in close contiguity to copper conductors never materially introduces self-induction flux-penetrations of the latter.* To prove this has been the object of the described experimentation herewith.

A slot three inches wide by four inches deep was cut in a sheet of galvanized iron. In the one case no flange was provided for the inner margin of the slot but in later experiments such flange was arranged without any material modifications in the character of the result obtained

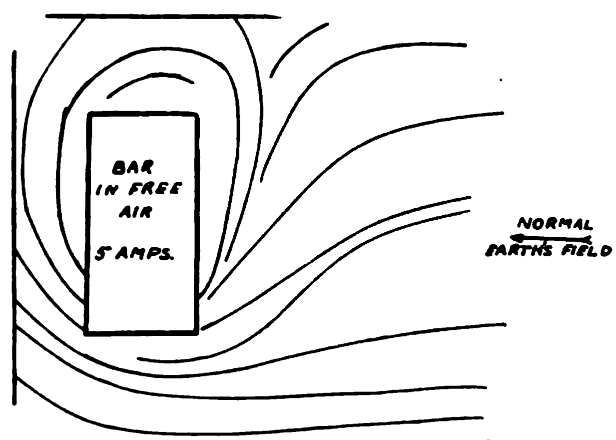

Fig. 1.

by the simpler arrangement. A bar of brass 2″ × 1″ as indicated in Fig. 2 was placed there in midway of the bar length of six inches so as to be one-half inch from the three walls of the slot. The plate and conductors were then arranged on a table and oriented so as to allow flux penetrations to occur relative to the three sides of the slot because of the presence of the earth's magnetic field. Such flux lines were explored and charted by means of a small watch charm compass of about five-sixteenths of an inch over-all diameter. The plate being horizontally arranged, the normal direction of the horizontal component of the earth's magnetic field is indicated by the arrow in Figs. 1 and 2. The thin lines of Fig. 2 represent the flux distribution in the slot with no current flowing in the brass conductor. It will be observed that the flux lines of the earth's magnetic field penetrate the conductor in their path toward the sides of the slot. In other words there is a decided lateral component parallel to the bottom of the slot, but there is no evidence of lines passing from one side of the slot directly across to the opposite side of the slot which latter effect might have been presumed to exist if the compass needle was able to induce sufficiently strong poles of opposite polarity in the two parallel walls of the slot. The diagram therefore

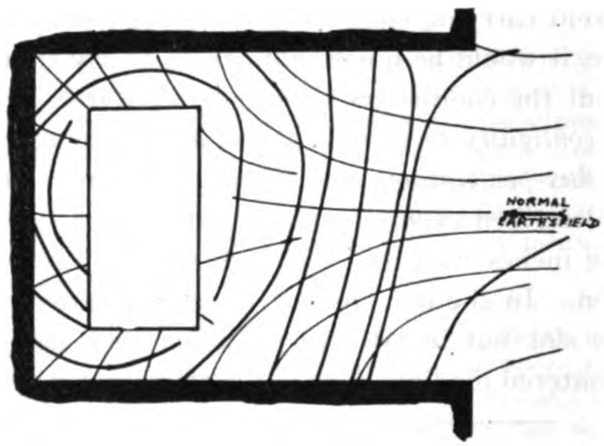

Fig. 2.

shows that with orientation settled upon the walls of the slot have substantially the same polarity due to the earth's magnetic field.

On passing a current of five amperes through the two square inches of brass the flux distribution in the slot radically changes. There is absolutely no evidence of lines of flux passing through the conductor from side to side to complete their circuit through the iron at the bottom of the slot. In fact a compass needle placed between the current carrying conductor and the sides of the slot is deflected so as to indicate that *the encircling lines actually pass up one side of the slot and down on the other.* Naturally, because of the presence of the earth's magnetic field a considerable skewing of flux lines occurs. The only place however where lines pass from side to side of the slot is above the conductor's surface. Between the conductor and the bottom of the slot lines do appear but they are parallel to the conductor and slot surfaces. As a control Fig. 1 was mapped out to show how the skewing of encircling flux lines occurs about the conductor due to the presence of the earth's magnetic field. In this case the same amount of current as before traversed the conductor.

Insofar as the effect of an alternating impressed E.M.F. upon a conductor is to crowd current toward the skin because of the self-induction flux linkages within the conductor, it is evident that according to the established theory mentioned in the opening paragraphs of this paper that such skin effect on the basis of short circuiting of self-induction lines by the iron of the slot would necessarily crowd the current to the top of the conductor within the slot. However, if the self-induction lines within the conductor are practically unaffected by the presence of iron masses about the conductor, it will mean in the case of rectangular conductors that there will be four boundary surfaces where the skin effect will take place instead of one as in the previous instance.

As a further check on the above theory coupled with the experimentation an improved mathematical investigation was made along classical lines. In the work of A. B. Field (loc. cit.) and others the following two conditions were imposed.

1. The magnetic potential $\int H \cdot dl$ beneath the mth conductor counting from the bottom of the slot was due to $(m - 1)$ conductors carrying the normal load current. This meant first, that only a lateral component of self-induction flux existed, and secondly, that beneath the first conductor no magnetic field was set up.

2. That the aggregate current in the mth conductor was equal to the load current.

It will be seen that no limitation was set as to the magnetic potential at the top of the mth conductor. In the improved mathematical investigation conducted by the writer the second condition was replaced by the following:

2*a*. The magnetic potential at the top of the mth conductor was due to m conductor carrying the line current.

In this way no necessity arose for separately indicating the line current in the mth conductor.

A solution of the differential equation leads to the following value of the average resistance factor K_M for an M layer winding would therefore be

$$K_M = a \cdot l \left\{ \frac{\sinh 2al + \sin 2al}{\cosh 2al - \cos 2al} + \tfrac{1}{2}(M - 1) \frac{\sinh al - \sin al}{\cosh al + \cos al} \right\}.$$

It is to be observed that the above value coincides with Field's finding on the assumption of a single layer winding only.[1]

Berkeley, California,
November 13, 1919.

[1] It is desired to acknowledge the help of my assistant Mr. Shuichi Sumioka in conducting the above experiments.